A simple multicloud parametrization for convectively coupled waves with an active boundary layer

Michael L. Waite and Boualem Khouider University of Victoria, Victoria, BC

Summary

- Intermediate models with crude vertical resolution are useful tools for the study of convectively coupled waves.
- Such models often employ a passive boundary layer (e.g. [2]).
- We incorporate active boundary layer equations into the Khouider-Majda multi-cloud model [2] by systematic averaging of Boussinesq equations through a layer of constant depth, following [6].
- New mechanisms for coupling of boundary layer and free troposphere: entrainment, divergence pumping
- Independent evolution of boundary layer θ provides natural framework for studying cloud–radiative feedback.
- Find large-scale band of linearly unstable convectively coupled waves, in agreement with [2].

Boundary layer equations

Reynolds equations for a Boussinesq boundary layer, **vertically averaged** through a **layer of constant uniform depth** h_b, are [6]

$$\frac{\mathsf{D}\boldsymbol{u}_b}{\mathsf{D}t} + f\hat{\boldsymbol{e}}_3 \times \boldsymbol{u}_b = -\frac{h_b}{2}\frac{g}{\theta_0}\boldsymbol{\nabla}\theta_b + \boldsymbol{\nabla}\cdot\boldsymbol{u}_b\,(\boldsymbol{u}_+ - \boldsymbol{u}_b) - \frac{1}{h_b}\langle w'\boldsymbol{u}'\rangle_+ + \frac{1}{h_b}\langle w'\boldsymbol{u}'\rangle_0,$$

$$\frac{\mathsf{D}\phi_b}{\mathsf{D}t} = \boldsymbol{\nabla}\cdot\boldsymbol{u}_b\,(\phi_+ - \phi_b) - \frac{1}{h_b}\left(\langle w'\phi'\rangle_+ - \langle w'\phi'\rangle_0\right) + S_b^{\phi},$$

where $D/Dt = \partial_t + \boldsymbol{u}_b\cdot\boldsymbol{\nabla}$; $\phi = \theta$, q or $\theta_e \equiv \theta + q$; subscript b denotes vertical average, 0 denotes evaluation at the surface, and $+$ denotes evaluation at $z = h_b^+$.

Assumptions:

- $\boldsymbol{\nabla}\cdot\langle\boldsymbol{u}'\boldsymbol{u}'\rangle = 0$, $\boldsymbol{\nabla}\cdot\langle\boldsymbol{u}'\phi'\rangle = 0$
- $(\langle\boldsymbol{u}\rangle\langle\boldsymbol{u}\rangle)_b = \boldsymbol{u}_b\,\boldsymbol{u}_b$, $(\langle\boldsymbol{u}\rangle\langle\phi\rangle)_b = \boldsymbol{u}_b\,\phi_b$
- Hydrostatic balance with $\theta = \theta_b$, negligible contribution of q to buoyancy

Closure

Close Reynolds stresses [4, 6]:

$$z = 0: \quad \langle w'\boldsymbol{u}'\rangle_0 \equiv C_d(u_0)\boldsymbol{u}_b, \quad \langle w'\phi'\rangle_0 \equiv \frac{h_b}{\tau_e}\Delta_0\phi,$$

$$z = h_b^+: \quad \langle w'\boldsymbol{u}'\rangle_+ \equiv \frac{h_b}{\tau_T}\Delta_+\boldsymbol{u}, \quad \langle w'\phi'\rangle_+ \equiv M_u\,\Delta_+\phi - M_d\,(\phi_m - \phi_+)\,,$$

where

$$\Delta_0\phi \equiv \phi_0 - \phi_b, \quad \Delta_+\phi \equiv \phi_b - \phi_+, \quad \Delta_m\phi \equiv \phi_b - \phi_m,$$

and subscript m denotes evaluation at the mid-troposphere.

Then

$$\frac{\mathsf{D}\phi_b}{\mathsf{D}t} = \frac{1}{\tau_e}\Delta_0\phi - \frac{E}{h_b}\Delta_+\phi - \frac{M_d}{h_b}\Delta_m\phi + S_b^{\phi},$$

where

$$E \equiv M + h_b\boldsymbol{\nabla}\cdot\boldsymbol{u}_b, \quad M \equiv M_u - M_d.$$

E is the **entrainment** across $z = h_b$, and M is the **net mass flux** out of the boundary layer (shallow updrafts minus deep downdrafts). Assume $M_d = \alpha_m M_u$ [4, 6], with $0 < \alpha_m \le 1$ specified. Following [2],

$$M_d \equiv \Lambda\frac{m_0}{\overline{P}}\left(\overline{P} + \mu_2(H_s - H_c)\right).$$

Write

$$\theta_+ \equiv \sigma_1\theta_1 + \sigma_2\theta_2, \quad q_+ \equiv \kappa q, \quad \boldsymbol{u}_+ \equiv \sqrt{2}(\boldsymbol{u}_1 + \boldsymbol{u}_2),$$

$$\theta_m \equiv \frac{2\sqrt{2}}{\pi}\left(\theta_1 + \alpha_2\theta_2\right), \quad q_m \equiv q.$$

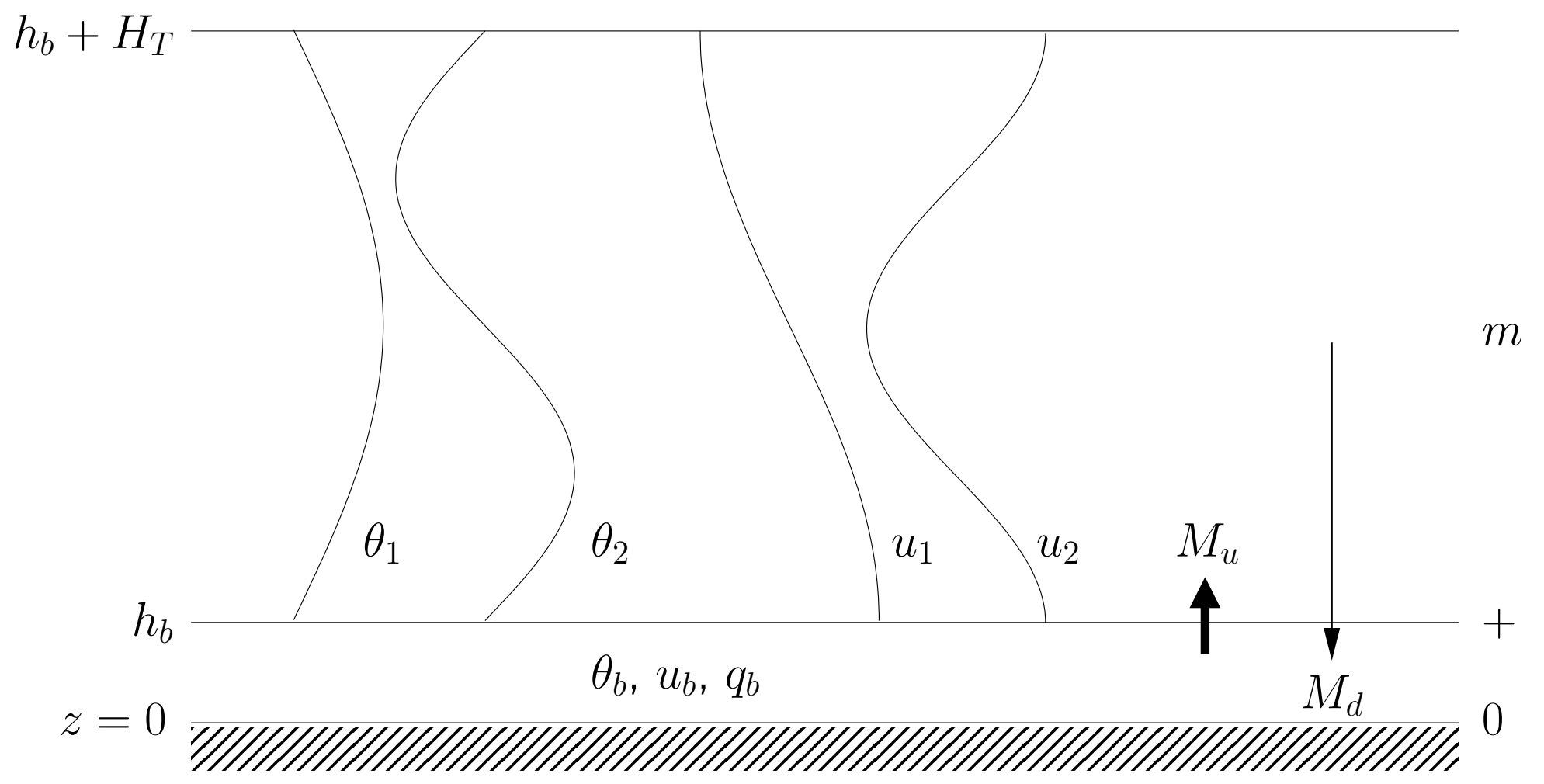
Introduce a divergent barotropic velocity $\boldsymbol{u}_0 = \boldsymbol{\nabla}\mathcal{X}$ to match boundary layer and free troposphere w at $z = h_b$ (following [1, 5]).

New parameters

$\alpha_m = 0.2$	Ratio of M_d to M_u
$\gamma = 0.5$	Ratio of $-\Delta_m\overline{\theta}$ to $\Delta_m\overline{\theta}_e$
$\kappa = 2$	Ratio of q_+ to q
$\sigma_2 = 0.1$	Contribution of θ_2 to θ_+; $\sigma_1 = \sigma_2/4$
$\alpha_1^d = 0.5$	Contribution of boundary layer drag to $\boldsymbol{u}_1$
$\tau_T \approx 3$ days	Momentum entrainment time scale
Q_{Rb}	Boundary layer radiative cooling rate; set by RCE
ν_0, ν_1, ν_2, $\nu_3 = 0$	Contribution of shallow cumulus, deep cumulus, stratiform and congestus clouds to cloud-radiative feedback

Standard parameter values used unless otherwise stated.

Multicloud model



A schematic of the multicloud model setup, comprising two baroclinic modes of depth H_T above a boundary layer of depth h_b.

Nondimensionalize following [2]; neglect baroclinic advection and Coriolis:

Boundary layer:

$$\partial_t\boldsymbol{u}_b = -\frac{\pi}{2}\frac{h_b}{H_T}\boldsymbol{\nabla}\theta_b - \left(\frac{1}{\tau_T} + \boldsymbol{\nabla}\cdot\boldsymbol{u}_b\right)\Delta_+\boldsymbol{u} - \frac{1}{h_b}C_d(u_0)\boldsymbol{u}_b,$$

$$\partial_t\theta_b = \frac{1}{\tau_e}\Delta_0\theta - \frac{E}{h_b}\Delta_+\theta - \frac{M_d}{h_b}\Delta_m\theta + S_b^{\theta},$$

$$\partial_t q_b = \frac{1}{\tau_e}\Delta_0 q - \frac{E}{h_b}\Delta_+ q - \frac{M_d}{h_b}\Delta_m q.$$

Free troposphere:

$$\partial_t\boldsymbol{u}_j + \boldsymbol{u}_0\cdot\boldsymbol{\nabla}\boldsymbol{u}_j - \boldsymbol{\nabla}\theta_j = -\frac{1}{\tau_R}\boldsymbol{u}_j + \left(\frac{1}{\tau_T} + \boldsymbol{\nabla}\cdot\boldsymbol{u}_b\right)\left(\frac{\alpha_j^d}{\sqrt{2}}\boldsymbol{u}_b - \boldsymbol{u}_j\right),$$

$$\partial_t\theta_1 + \boldsymbol{u}_0\cdot\boldsymbol{\nabla}\theta_1 - \boldsymbol{\nabla}\cdot\boldsymbol{u}_1 = \frac{\pi}{2\sqrt{2}}P - Q_{R1}^0 - \frac{1}{\tau_D}\theta_1,$$

$$\partial_t\theta_2 + \boldsymbol{u}_0\cdot\boldsymbol{\nabla}\theta_2 - \frac{1}{4}\boldsymbol{\nabla}\cdot\boldsymbol{u}_2 = \frac{\pi}{2\sqrt{2}}(-H_s + H_c) - Q_{R2}^0 - \frac{1}{\tau_D}\theta_2,$$

$$\partial_t q + \boldsymbol{u}_0\cdot\boldsymbol{\nabla}q + \boldsymbol{\nabla}\cdot\left([\boldsymbol{u}_1 + \tilde{\delta}\boldsymbol{u}_2]\,q\right) + \tilde{Q}\,\boldsymbol{\nabla}\cdot(\boldsymbol{u}_1 + \tilde{\lambda}\boldsymbol{u}_2) = -P + \frac{E}{H_T}\Delta_+\theta_e + \frac{M_d}{H_T}\Delta_m\theta_e,$$

$$\partial_t H_s = \frac{1}{\tau_s}(\alpha_s P - H_s), \quad \partial_t H_c = \frac{1}{\tau_c}\left(\alpha_c\left[\frac{\Lambda - \Lambda^*}{1 - \Lambda^*}\right]\frac{M_d}{H_T}\Delta_m\theta_e - H_c\right),$$

where

$$P \equiv \frac{1 - \Lambda}{1 - \Lambda^*}\frac{1}{\tau_{\mathrm{conv}}}\left(a_1\theta_{eb} + a_2(q - \widehat{q}) - a_0(\theta_1 + \gamma_2\theta_2)\right)^+,$$

$$\Lambda \equiv \begin{cases} 1 & \text{if} \quad \Delta_m\theta_e > \theta^+, \\ A(\theta_{eb} - \theta_{em}) + B & \text{if} \quad \theta^- < \Delta_m\theta_e < \theta^+, \\ \Lambda^* & \text{if} \quad \Delta_m\theta_e < \theta^-, \end{cases}$$

$$\boldsymbol{u}_0 = \boldsymbol{\nabla}\mathcal{X}, \quad \nabla^2\mathcal{X} = -\frac{h_b}{H_T}\boldsymbol{\nabla}\cdot\boldsymbol{u}_b,$$

$$S_b^{\theta} \equiv -Q_{Rb} + \nu_0\frac{\overline{\alpha}}{\alpha_m}\frac{M_d}{h_b} + \nu_1 P + \nu_2 H_s + \nu_3 H_c, \quad S_b^{q} \equiv 0.$$

Parameters (see [2])

$H_T = 16$ km
$h_b = 500$ m
$\tau_R = 75$ days
$\tau_D = 50$ days
$\tau_s = 3$ hours
$\tau_c = 1$ hours
$\tau_{conv} = 2$ hours
$\overline{\alpha} \approx 15$ K

$\alpha_s = 0.25$
$\alpha_c = 0.5$
$a_0 = 7.5$
$a_1 = 0.1$
$a_2 = 0.9$
$\gamma_2 = 0.1$
$\mu_2 = 0.5$

What's new?

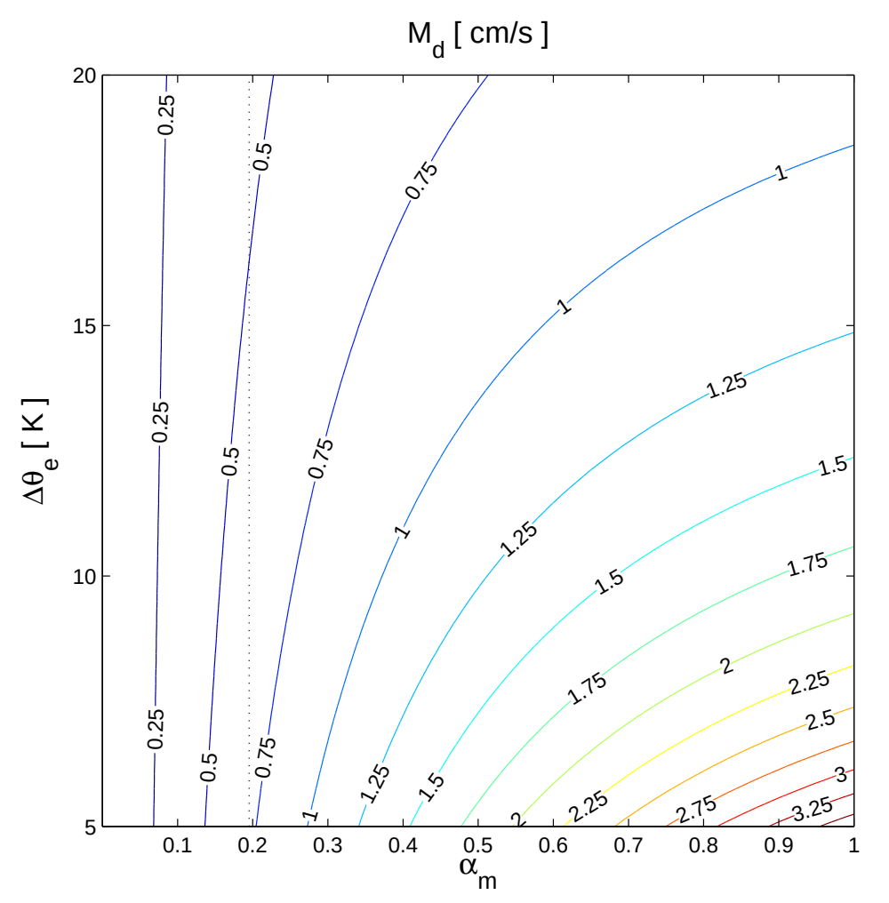
Compared to [2], this model has:

- **Independent** evolution of θ_b and q_b
- **Advection of** θ_b and q_b
- **Entrainment** across $z = h_b$
- **Pumping** of free troposphere by $\boldsymbol{\nabla}\cdot\boldsymbol{u}_b$ through $\boldsymbol{u}_0$
- **Cloud–radiative feedback** through parameters ν_j

Radiative–convective equilibrium

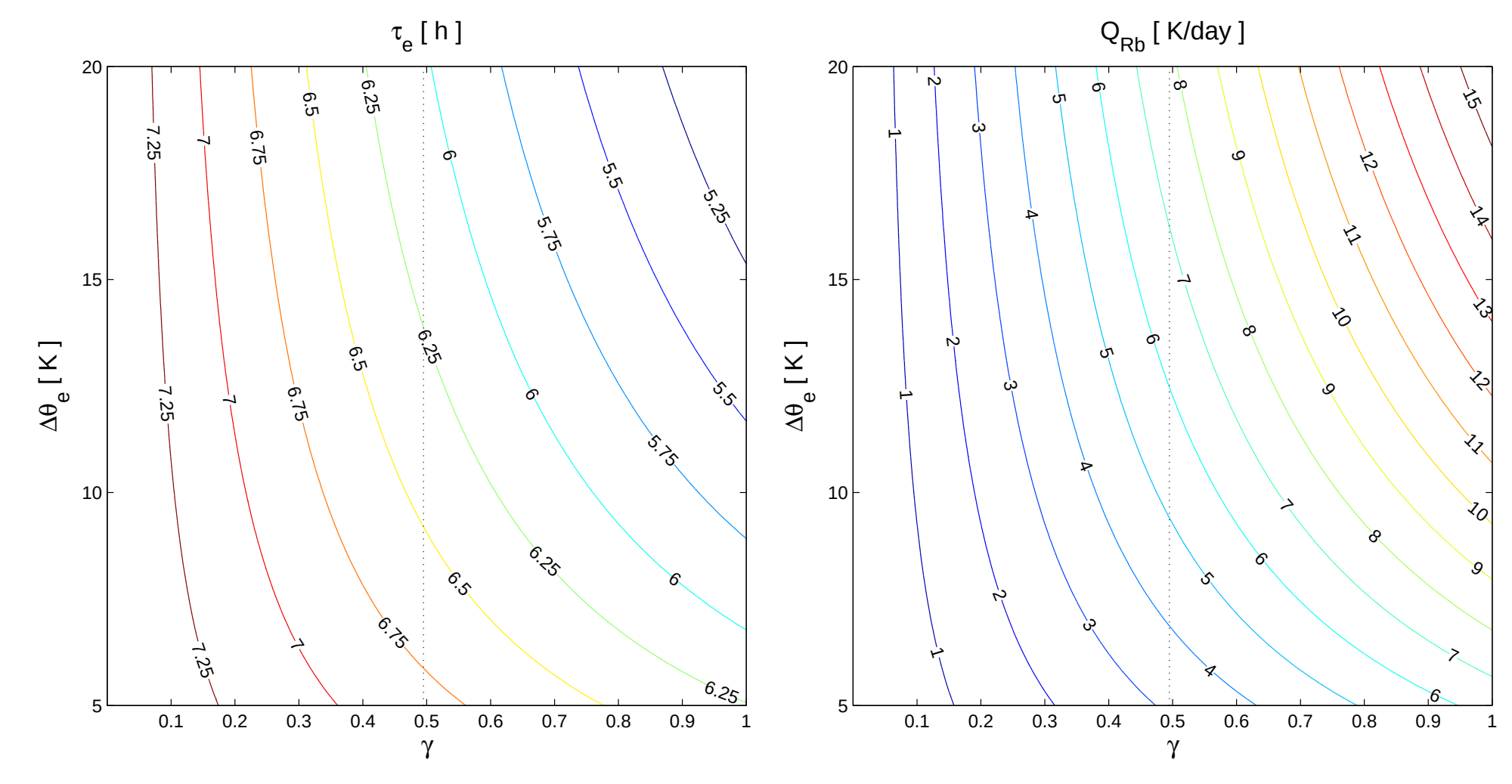
Radiative–convective equilibrium (RCE), denoted by $\overline{(\cdot)}$, is determined by specifying radiative cooling and background thermodynamic state:

$$\begin{array}{lll} \tilde{Q}_{R1} = 1\ \text{K/day}\ , & \Delta_0\overline{\theta} = 0\ \text{K}\ , & \Delta_m\overline{\theta} = -\gamma\Delta_m\overline{\theta}_e\ , \\ & \Delta_0\overline{q} = 10\ \text{K}\ , & \Delta_m\overline{q} = (1+\gamma)\Delta_m\overline{\theta}_e\ , \\ & \Delta_+\overline{\theta} = 0\ \text{K}\ , & \Delta_m\overline{\theta}_e\ \text{varies}\ . \\ & \Delta_+\overline{q} = 5K\ , & \end{array}$$



For $\Delta_m\overline{\theta}_e = 14$ K,

- $\alpha_m = 0.2$ yields $M_d \approx 0.5$ cm/s.
- $\gamma = 0.5$ yields
  - $\Delta_m\overline{\theta} = -7$ K,
  - $\Delta_m\overline{q} \approx 8$ g/kg,
  - $\tau_e \approx 6$ hours (c.f. [2]),
  - $Q_{Rb} \approx 7$ K/day (cf. [3]).



The RCE downward mass flux velocity $\overline{M}_d$ (top left), evaporative time scale τ_e (bottom left), and boundary layer radiative cooling rate Q_{Rb} (bottom right), as a function of $\Delta_m\overline{\theta}_e$ and α_m (for $\overline{M}_d$) or γ (for τ_e and Q_{Rb}, assuming $\alpha_m = 0.2$). The standard values are marked with a dotted line.

References

[1] J. A. Biello and A. J. Majda. Boundary layer dissipation and the nonlinear interaction of equatorial baroclinic and barotropic Rossby waves. *Geophys. Astrophys. Fluid Dyn.*, 98:85–127, 2004.

[2] B. Khouider and A. J. Majda. A simple parameterization for convectively coupled tropical waves. Part I: Linear analysis. *J. Atmos. Sci.*, 63:1308–1323, 2006.

[3] E. B. Kraus and H. P. Hanson. Climatological aspects of the tropical convective boundary layer. *Boundary-Layer Meteorol.*, 6:219–233, 1974.

[4] D. J. Raymond. Regulation of moist convection over the West Pacific warm pool. *J. Atmos. Sci.*, 52:3945–3959, 1995.

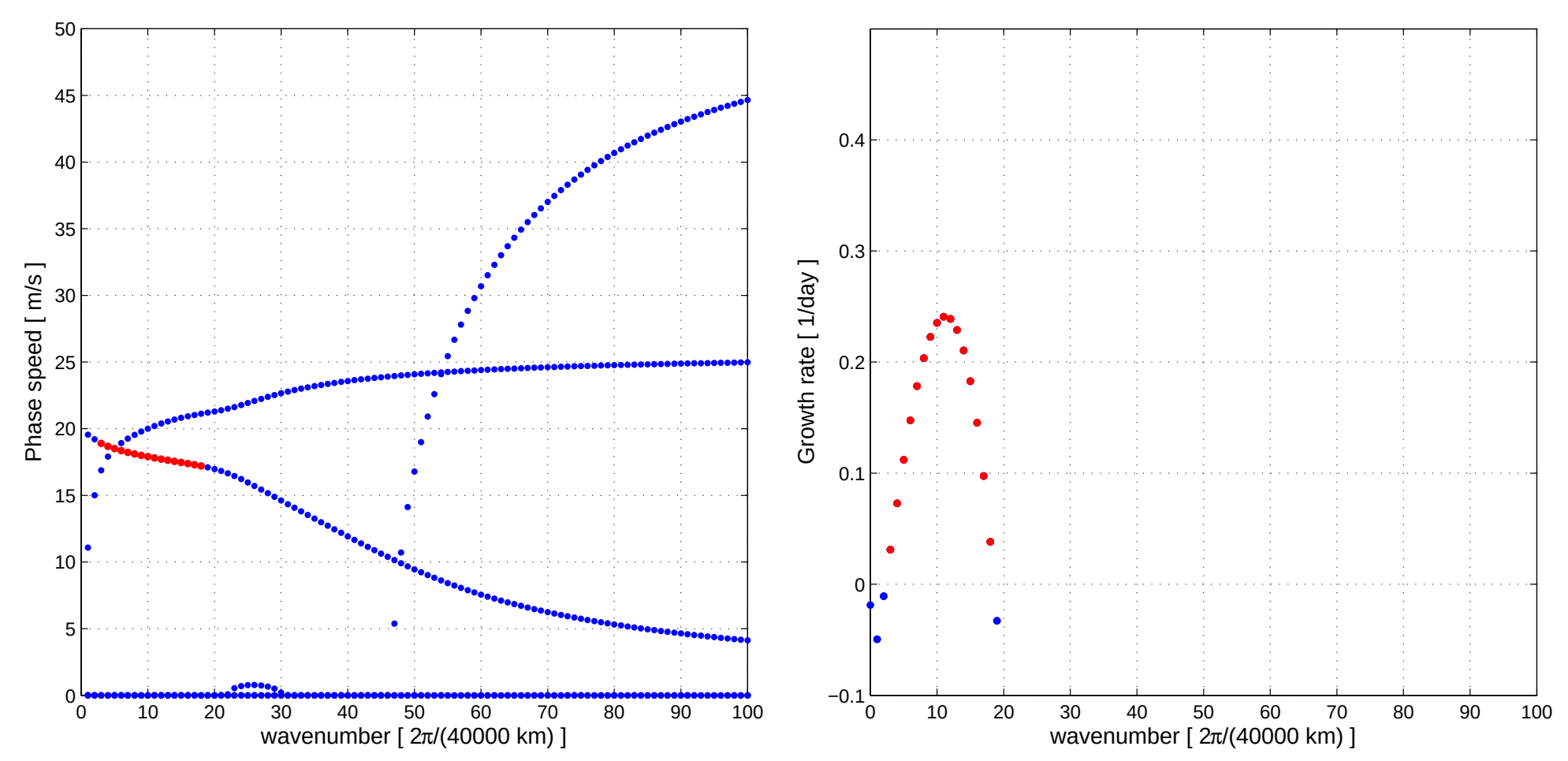
[5] A. H. Sobel and J. D. Neelin. The boundary layer contribution to intertropical convergence zones in the quasi-equilibrium tropical circulation model framework. *Theor. Comput. Fluid Dyn.*, 20:323–350, 2006.

[6] B. Stevens. Bulk boundary-layer concepts for simplified models of tropical dynamics. *Theor. Comput. Fluid Dyn.*, 20:279–304, 2006.

Linear stability

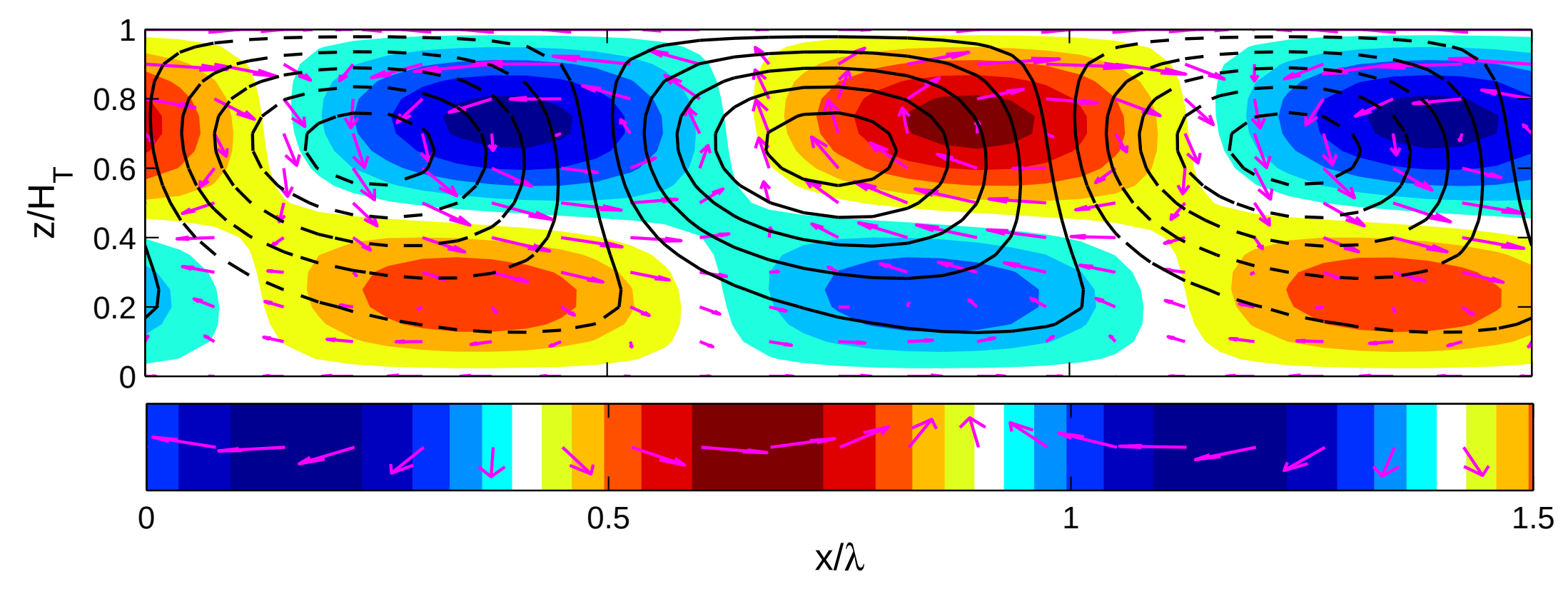
We linearize around the RCE, and compute phase speeds and growth rates. The standard parameter values are well inside the region of stability to homogeneous (wavenumber $k = 0$) perturbations.

For a mixed deep-convective/congestus RCE with $\Delta_m\overline{\theta}_e = 14$ K, there is a band of unstable waves with phase speeds ≈ 15–20 m/s (below left) and a most unstable wavelength ≈ 4000 km (below right). The inclusion of an active boundary layer does not qualitatively change the basic instability described by [2].

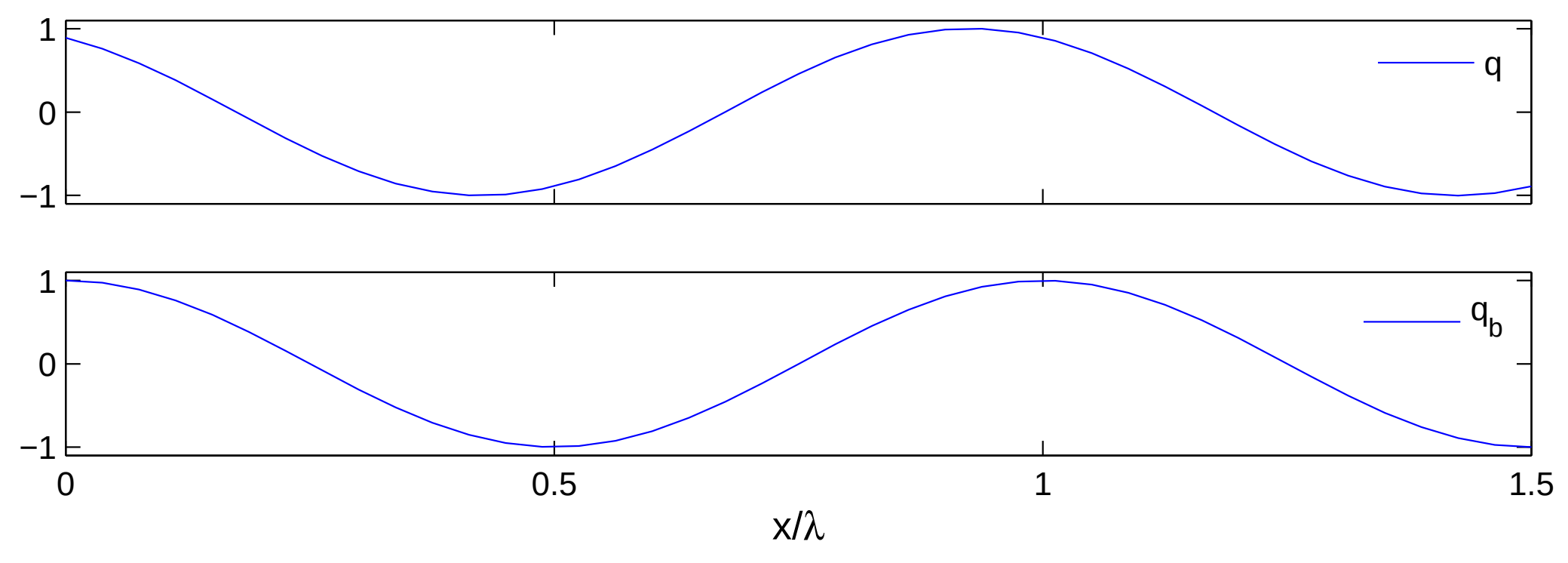


Phase speeds (left) and growth rates (right) as a function of wavenumber. Unstable modes are plotted in red.

The structure of the most unstable wave (below, with wavenumber 11, or wavelength $\lambda = 3636$ km) resembles that of [2]. Upper troposphere positive temperature anomalies lead the heating and updrafts, and the wave exhibits a westward tilt with height. The boundary layer temperature and moisture leads that in the lower free troposphere.



Physical structure of the most unstable wave in the free troposphere (top) and boundary layer (bottom). Potential temperature is shaded (warm temperatures in red, cool in blue), total convective heating is contoured (negative values dashed), and flow vectors are superimposed.



Average moisture in the free-troposphere (top) and boundary layer (bottom).

Acknowledgements

Thanks to Andrew Majda and Bjorn Stevens. Financial support provided by the Natural Sciences and Engineering Research Council of Canada and the Pacific Institute for the Mathematical Sciences.